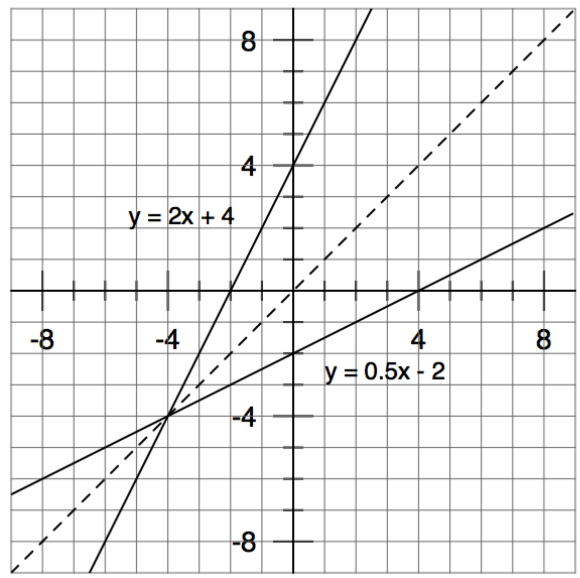
Inverse Relations

These notes are intended as a summary of section 3.5 (p. 235 – 242) in your workbook. You should also read the section for more complete explanations and additional examples.

The graphs of $y = 2x + 4$ and $y = \frac{1}{2}x - 2$ are shown on the axes to the right. The graph of $y = \frac{1}{2}x - 2$ is the graph of $y = 2x + 4$ after a reflection in the line $y = x$.



Consider the following points on each graph:

Point on $y = 2x + 4$	**Point on** $y = \frac{1}{2}x - 2$
$(1,6)$	$(6,1)$
$(0,4)$	$(4,0)$
$(-2,0)$	$(0,-2)$

Notice that the coordinates of corresponding points are interchanged. $y = \frac{1}{2}x - 2$ is the **inverse** of the function $y = 2x + 4$.

Reflecting in the Line $y = x$

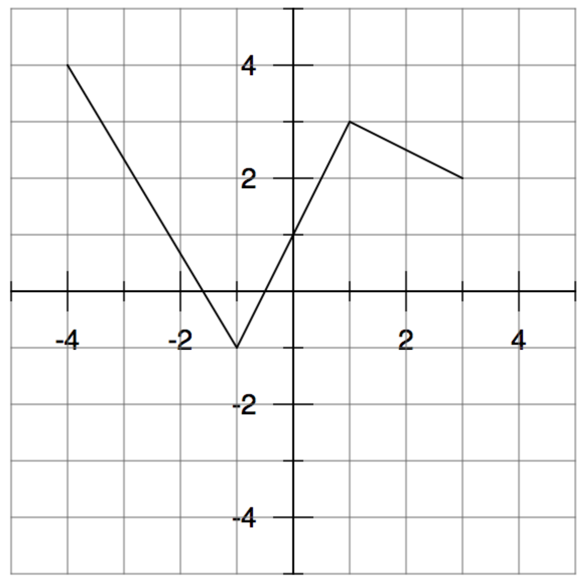
In general, the graph of $x = f(y)$ is the image of the graph of $y = f(x)$ after a reflection in the line $y = x$. $y = f(x)$ and $x = f(y)$ are said to be **inverses** of each other.

For every point (x,y) on $y = f(x)$, there is a corresponding point (y,x) on $x = f(y)$.

When the inverse is also a function, the notation $f^{-1}(x)$ is used to denote the **inverse function**. For example, when $f(x) = 2x + 4$, then $f^{-1}(x) = \frac{1}{2}x - 2$.

Note: To determine if the inverse is a function, use the **vertical line test**.

Example 1 (sidebar p. 237)
Here is the graph of $y = g(x)$.



a) Sketch the graph of its inverse on the same grid.

b) Is the inverse a function? Explain.

c) State the domain and range of the function and its inverse.

Domain and Range

The domain of $y = f(x)$ is the range of $x = f(y)$, and the range of $y = f(x)$ is the domain of $x = f(y)$.

Determining an Equation of the Inverse

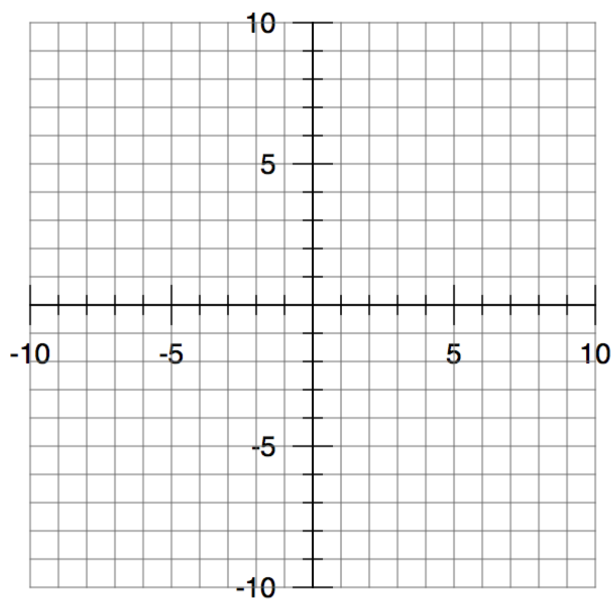
To determine an equation for the inverse of a function, interchange *x* and *y* in the equation of the function, then solve the resulting equation for *y*.

For example, to determine the equation of the inverse of $y = -3x + 7$:

Example 2 (sidebar p. 238)

a) Determine an equation of the inverse of $y = -x^2 + 4$.

b) Sketch graphs of $y = -x^2 + 4$ and its inverse.



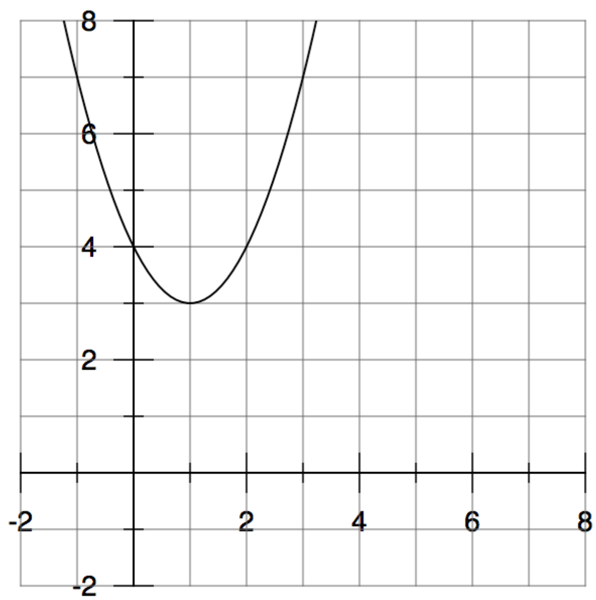
c) Is the inverse a function? Explain.

Think Further (bottom of p. 239)
When is the inverse of a function also a function?

It is possible to predict whether or not an inverse of a function will also be a function. To do so, perform a horizontal line test on the original function. If the original function passes the horizontal line test, then its inverse will be a function. If the original function fails the horizontal line test, then its inverse won't be a function.

Example 3 (sidebar p. 240)

Determine two ways to restrict the domain of $y=(x-1)^2+3$ so that its inverse is a function. Write the equation of the inverse each time. Use a graph to illustrate each way.



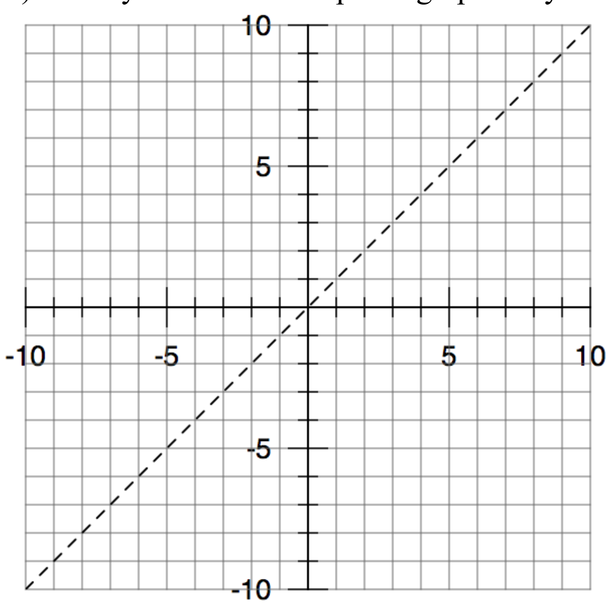
Example 4 (sidebar p. 241)

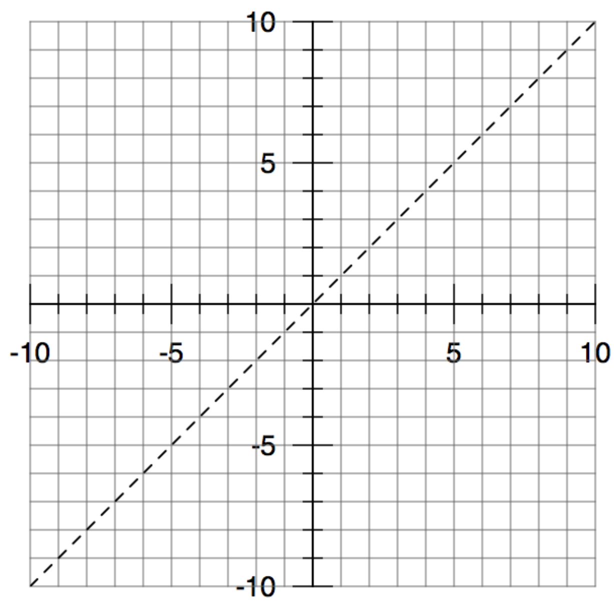
a) Determine algebraically whether the functions in each pair are inverses of each other.

i) $y = 3x - 6$ and $y = \dfrac{x-6}{3}$

ii) $y = -x^2 + 3$, $x \geq 0$ and $y = \sqrt{3 - x}$

b) Verify the answers to part a graphically.





Homework: #4 – 13 in the exercises (p. 243 – 249). Answers on p. 250.